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# STABILITY ANALYSIS OF MATHEMATICAL SYN-ECOLOGICAL MODEL COMPRISING OF PREY-PREDATOR, HOST-COMMENSAL, MUTUALISM AND NEUTRAL PAIRS-II

## (THREE OF THE FOUR SPECIES ARE WASHED OUT STATES)

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***Abstract***

This investigation deals with a mathematical model of a four species ($S_1$, $S_2$, $S_3$ and $S_4$) Syn-Ecological system (Three of the four species are washed out states). $S_2$ is a predator surviving on the prey $S_1$. The predator $S_2$ is a commensal to the host $S_3$. The pairs $S_2$ and $S_4$, $S_1$ and $S_3$ are neutral. The mathematical model equations characterizing the syn-ecosystem constitute a set of four first order non-linear coupled differential equations. There are in all sixteen equilibrium points. Criteria for the asymptotic stability of four of the sixteen equilibrium points: Three of the four species are washed out states only are established in this paper. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories illustrated.

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## 1. INTRODUCTION

Mathematical modeling is an important interdisciplinary activity which involves the study of some aspects of diverse disciplines. Biology, Epidemiodology, Physiology, Ecology, Immunology, Bio-economics, Genetics, Pharmocokinetics are some of those disciplines. This mathematical modeling has raised to the zenith in recent years and spread to all branches of life and drew the attention of every one. Mathematical modeling of ecosystems was initiated by Lotka [9] and by Volterra [18]. The general concept of modeling has been presented in the treatises of Meyer [11], Cushing [4], Paul Colinvaux [11], Freedman [5], Kapur [6, 7]. The ecological interactions can be broadly classified as Prey-Predation, Competition, Mutualism and so on. N.C. Srinivas [17] studied the competitive eco-systems of two species and three species with regard to limited and unlimited resources. Later, Lakshmi Narayan [8] has investigated the two species prey-predator models. Stability analysis of competitive species was carried out by Archana Reddy [3] while Acharyulu [1, 2] investigated Ammensalism between two species. Recently local stability analysis for a two-species ecological mutualism model has been investigated by present author et al [12, 13, 14, 15, 16]. Example for $S_1$, $S_2$, $S_3$ and $S_4$ are Insects, Insectivorous Plants (nephantis, drosera etc.), VAM associated with the plant roots, Soil bacteria respectively.

## 2. BASIC EQUATIONS

The model equations for a four species multi-system are given by a set of four non-linear ordinary differential equations as

(i) For $S_1$**:** The Prey of $S_1$ and Neutral to $S_3$

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 \qquad (2.1)$$

(ii) For $S_2$**:** The Predator surviving on $S_1$ and Commensal to $S_3$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 + a_{21}N_2N_1 + a_{23}N_2N_3 \qquad (2.2)$$

(iii) For $S_3$**:** The Host of $S_2$ and Mutual to $S_4$

$$\frac{dN_3}{dt} = a_3N_3 - a_{33}N_3^2 + a_{34}N_3N_4 \qquad (2.3)$$

(iv) For $S_4$**:** Mutual to $S_3$ and Neutral to $S_2$

$$\frac{dN_4}{dt} = a_4N_4 - a_{44}N_4^2 + a_{43}N_4N_3 \qquad (2.4)$$

with the following notation.

$N_i$ (t): Population strengths of the species $S_i$ at time t, i=1, 2, 3, 4.
$a_i$ : The natural growth rates of $S_i$, i = 1,2,3,4
$a_{12}$,$a_{21}$ : Interaction (Prey-Predator) coefficients of $S_1$ due to $S_2$ and $S_2$ due to $S_1$
$a_{13}$ : Coefficient for commensal for $S_1$ due to the Host $S_3$
$a_{34}$, $a_{43}$ : Mutually interaction between $S_3$ and $S_4$
$K_i$: $\frac{a_i}{a_{ii}}$ : Carrying capacities of $S_i$, i=1, 2, 3, 4.

Further the variables $N_1$, $N_2$, $N_3$, $N_4$ are non-negative and the model parameters $a_1$, $a_2$, $a_3$, $a_4$; $a_{11}$, $a_{22}$, $a_{33}$, $a_{44}$; $a_{12}$, $a_{21}$, $a_{13}$, $a_{24}$ are assumed to be non-negative constants.

## 3. EQUILIBRIUM STATES

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \qquad (3.1)$$

are given in the following table.

**I.** Fully washed out state:

E1: $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$

**II.** States in which three of the four species are washed out and fourth is surviving

E2: $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

E3: $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

E4: $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$

E5: $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$

**III.** States in which two of the four species are washed out while the other two are surviving

E6: $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33}a_{44} - a_{34}a_{43}}, \overline{N_4} = \frac{a_3 a_{43} + a_4 a_{33}}{a_{33}a_{44} - a_{34}a_{43}}$

This state exists only when $a_{33}a_{44} - a_{34}a_{43} > 0$

E7: $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

E8: $\overline{N_1} = 0, \overline{N_2} = \frac{a_3}{a_{22}}\frac{a_{23}}{a_{33}} + \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

E9: $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

E10: $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

E11: $\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11}a_{22} + a_{12}a_{21}}, \overline{N_2} = \frac{a_1 a_{21} + a_2 a_{11}}{a_{11}a_{22} + a_{12}a_{21}}, \overline{N_3} = 0, \overline{N_4} = 0$

This state exists only when $a_1 a_{22} - a_2 a_{12} > 0$

**IV.** States in which one of the four species is washed out while the other three are surviving

E12: $\overline{N_1} = 0, \overline{N_2} = \frac{a_{23}(a_4 a_{34} + a_3 a_{44})}{a_{22}(a_{33}a_{44} - a_{34}a_{43})} + \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33}a_{44} - a_{34}a_{43}},$

$\overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33}a_{44} - a_{34}a_{43}}$

This state exists only when $a_{33}a_{44} - a_{34}a_{43} > 0$

E13: $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33}a_{44} - a_{34}a_{43}}, \overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33}a_{44} - a_{34}a_{43}}$

This state exists only when $(a_{33}a_{44} - a_{34}a_{43}) > 0$

E14: $\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11}a_{22} + a_{12}a_{21}}, \overline{N_2} = \frac{a_1 a_{21} + a_2 a_{11}}{a_{11}a_{22} + a_{12}a_{21}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

This state exists only when $a_1 a_{22} - a_2 a_{12} > 0$

E15: $\overline{N_1} = \frac{\beta_4}{\beta_1}, \overline{N_2} = \frac{\beta_5}{\beta_1}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$

where

$\beta_1 = a_{33}(a_{11}a_{22} + a_{12}a_{21}),\ \beta_4 = a_{33}(a_1 a_{22} - a_2 a_{12}) - a_3 a_{23} a_{12}$

$\beta_5 = a_{33}(a_1 a_{21} + a_2 a_{11}) + a_3 a_{23} a_{11}$

This state exists only when $\beta_4 > 0$

**V.** The co-existent state (or) Normal steady state

E16: $\overline{N_1} = \frac{\gamma_1 + a_{12}a_{23}\gamma_2}{\gamma_3(a_{33}a_{44} - a_{34}a_{43})}, \overline{N_2} = \frac{\gamma_4 + a_{11}a_{23}\gamma_2}{\gamma_3(a_{33}a_{44} - a_{34}a_{43})},$

$\overline{N_3} = \frac{a_4 a_{34} + a_3 a_{44}}{a_{33}a_{44} - a_{34}a_{43}}, \overline{N_4} = \frac{a_4 a_{33} + a_3 a_{43}}{a_{33}a_{44} - a_{34}a_{43}}$

Where

$\gamma_1 = (a_1 a_{22} + a_2 a_{12})(a_{33}a_{44} - a_{34}a_{43}),\ \gamma_2 = a_3 a_{44} + a_4 a_{34}$

$\gamma_3 = a_{11}a_{22} + a_{12}a_{21},\ \gamma_4 = (a_1 a_{21} - a_2 a_{11})(a_{33}a_{44} - a_{34}a_{43})$

This state exists only when $(a_1 a_{21} - a_2 a_{11}) > 0$ *and* $(a_{33}a_{44} - a_{34}a_{43}) > 0$.

The present paper deals with three of the four species are washed out states only. The stability of the other equilibrium states will be presented in the forth coming communications.

## 4. STABILITY OF THREE OF THE FOUR SPECIES WASHED OUT EQUILIBRIUM STATES
**(Sl. Nos 2,3,4,5 in the above Equilibrium States)**

### 4.1 Stability of the Equilibrium State $E_2$:

$$\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}} \qquad (4.1.1)$$

Substituting (4.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of $u_1$, $u_2$, $u_3$, $u_4$, we get

$$\frac{du_1}{dt}=a_1u_1 \qquad (4.1.2) \qquad \frac{du_2}{dt}=a_2u_2 \qquad (4.1.3)$$

$$\frac{du_3}{dt}=l_3u_3 \qquad (4.1.4) \qquad \frac{du_4}{dt}=-a_4u_4+\frac{a_{43}a_4}{a_{44}}u_3 \qquad (4.1.5)$$

Here $l_3=a_3+\frac{a_{34}a_4}{a_{44}}$ (4.1.6)

The characteristic equation of which is

$$(\lambda-a_1)(\lambda-a_2)(\lambda-l_3)(\lambda+a_4)=0 \qquad (4.1.7)$$

The roots $a_1$, $a_2$, $l_3$ are positive and $-a_4$ is negative.

Hence the steady state is **unstable**.

The solutions of the equations (4.1.2), (4.1.3), (4.1.4), (4.1.5) are

$$u_1=u_{10}e^{a_1t} \qquad (4.1.8)$$

$$u_2=u_{20}e^{a_2t} \qquad (4.1.9)$$

$$u_3=u_{30}e^{l_3t} \qquad (4.1.10)$$

$$u_4=[u_{40}-\frac{a_{43}a_4u_{30}}{a_{44}(l_3+a_4)}]e^{-a_4t}+\frac{a_{43}a_4u_{30}}{a_{44}(l_3+a_4)}e^{l_3t} \qquad (4.1.11)$$

where $u_{10}$, $u_{20}$, $u_{30}$, $u_{40}$ are the initial values of $u_1$, $u_2$, $u_3$, $u_4$ respectively.

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates $a_1$, $a_2$, $a_3$, $a_4$ and the initial values of the perturbations $u_{10}(t)$, $u_{20}(t)$, $u_{30}(t)$, $u_{40}(t)$ of the species $S_1$, $S_2$, $S_3$, $S_4$. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations.

The solutions are illustrated in figures.

**Case (i):** If $u_{30} < u_{40} < u_{20} < u_{10}$ and $a_3 < a_2 < a_4 < a_1$

In this case initially $S_4$ dominates the Predator ($S_2$) and the host ($S_3$) of $S_2$ till the time instant $t^*_{24}$, $t^*_{34}$ respectively and thereafter the dominance is reversed. Also the Predator ($S_2$) dominates the host ($S_3$) of $S_2$ till the time instant $t^*_{32}$ and thereafter the dominance is reversed.

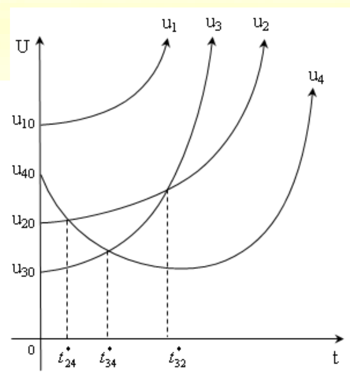
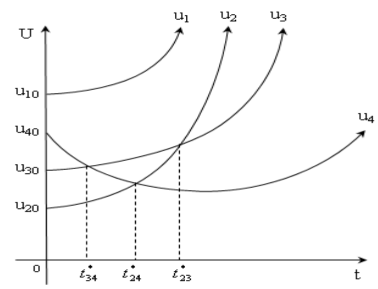



Fig. 1

**Case (ii):** If $u_{20} < u_{30} < u_{40} < u_{10}$ and $l_3 < a_2 < a_1 < a_4$




In this case initially $S_4$ dominates the host ($S_3$) of $S_2$ and the Predator ($S_2$) till the time instant $t^*_{34}$, $t^*_{24}$ respectively and thereafter the dominance is reversed. Also the host ($S_3$) of $S_2$ dominates the Predator ($S_2$) till the time instant $t^*_{23}$ and thereafter the dominance is reversed.

Fig. 2

### 4.2 Stability of the Equilibrium State E₃:

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of $u_1$, $u_2$, $u_3$, $u_4$, we get

$$\frac{du_1}{dt} = a_1 u_1 \qquad (4.2.1) \qquad\qquad \frac{du_2}{dt} = m_2 u_2 \qquad (4.2.2)$$

$$\frac{du_3}{dt} = -a_3 u_3 + \frac{a_{34} a_3}{a_{33}} u_4 \qquad (4.2.3) \qquad\qquad \frac{du_4}{dt} = n_4 u_4 \qquad (4.2.4)$$

Here $m_2 = a_2 + \frac{a_{23} a_3}{a_{33}}$, $n_4 = a_4 + \frac{a_{43} a_3}{a_{33}}$

The characteristic equation of which is

$$(\lambda - a_1)(\lambda - m_2)(\lambda + a_3)(\lambda - n_4) = 0 \qquad (4.2.5)$$

The roots $a_1$, $m_2$, $n_4$ are positive and $-a_3$ is negative.

Hence the steady state is **unstable**.

The solutions of the equations (4.2.1), (4.2.2), (4.2.3), (4.2.4) are

$$u_1 = u_{10} e^{a_1 t} \qquad (4.2.6)$$

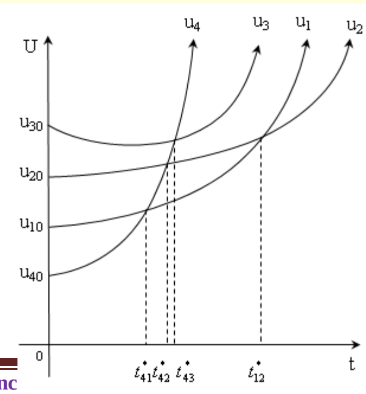
$$u_2 = u_{20} e^{m_2 t} \qquad (4.2.7)$$

$$u_3 = [u_{30} - \frac{a_{34} a_3 u_{40}}{a_{33}(n_4 + a_3)}] e^{-a_3 t} + \frac{a_{34} a_3 u_{40}}{a_{33}(n_4 + a_3)} e^{n_4 t} \qquad (4.2.8)$$

$$u_4 = u_{40} e^{n_4 t} \qquad (4.2.9)$$

The solutions are illustrated in figures.

**Case (i):** If $u_{40} < u_{10} < u_{20} < u_{30}$ and $m_2 < a_3 < a_1 < n_4$




In this case initially the host ($S_3$) of $S_2$ dominates $S_4$ till the time instant $t^*_{43}$ and thereafter the dominance is reversed. Also the Predator ($S_2$) dominates the Prey ($S_1$) and $S_4$ till the time instant $t^*_{12}$, $t^*_{42}$ respectively and the dominance gets reversed thereafter. Similarly the Prey ($S_1$) dominates $S_4$ till the time instant $t^*_{41}$ and thereafter the dominance is reversed.

Fig. 3

**Case (ii):** If $u_{10} < u_{20} < u_{30} < u_{40}$ and $n_4 < a_1 < a_3 < m_2$

In this case initially $S_4$ dominates the Predator ($S_2$) till the time instant $t^*_{24}$ and thereafter the dominance is reversed. Also the host ($S_3$) of $S_2$ dominates the Predator ($S_2$) and the Prey ($S_1$) till the time instant $t^*_{23}$, $t^*_{13}$ respectively and the dominance gets reversed thereafter.

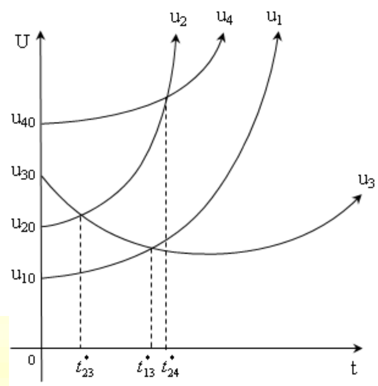



Fig. 4

### 4.3 Stability of the Equilibrium State $E_4$:

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of $u_1$, $u_2$, $u_3$, $u_4$, we get

$$\frac{du_1}{dt} = r_1 u_1 \qquad (4.3.1) \qquad \frac{du_2}{dt} = -a_2 u_2 + \frac{a_{21}a_2}{a_{22}}u_1 + \frac{a_{23}a_2}{a_{22}}u_3 \qquad (4.3.2)$$

$$\frac{du_3}{dt} = a_3 u_3 \qquad (4.3.3) \qquad \frac{du_4}{dt} = a_4 u_4 \qquad (4.3.4)$$

Here $r_1 = a_1 - \frac{a_{12}a_2}{a_{22}}$ (4.3.5)

The characteristic equation of which is

$$(\lambda - r_1)(\lambda + a_2)(\lambda - a_3)(\lambda - a_4) = 0 \qquad (4.3.6)$$

**Case (A):** When $r_1 < 0$ (i.e., when $\frac{a_1}{a_2} < \frac{a_{12}}{a_{22}}$)

The roots $r_1, -a_2$ are negative and $a_3, a_4$ are positive.

Hence the equilibrium state is **unstable.**

The solutions of the equations (4.3.1) (4.3.2), (4.3.3), (4.3.4) are

$$u_1 = u_{10}e^{r_1 t} \qquad (4.3.7)$$

$$u_2 = \left\{ u_{20} - \left[\frac{a_{21}a_2u_{10}}{a_{22}(r_1 + a_2)} + \frac{a_{23}a_2u_{30}}{a_{22}(a_2 + a_3)}\right]\right\} e^{-a_2 t} + \frac{a_{21}a_2u_{10}}{a_{22}(r_1 + a_2)}e^{r_1 t} + \frac{a_{23}a_2u_{30}}{a_{22}(a_2 + a_3)}e^{a_3 t} \qquad (4.3.8)$$

$$u_3 = u_{30}e^{a_3 t} \qquad (4.3.9)$$

$$u_4 = u_{40} e^{a_4 t} \qquad (4.3.10)$$

The solutions are illustrated in figures.

**Case (i):** If $u_{20} < u_{40} < u_{10} < u_{30}$ and $a_4 < a_3 < a_2 < r_1$

In this case initially the Prey ($S_1$) dominates $S_4$ and the Predator ($S_2$) till the time instant $t^*_{41}$, $t^*_{21}$ respectively and thereafter the dominance is reversed.

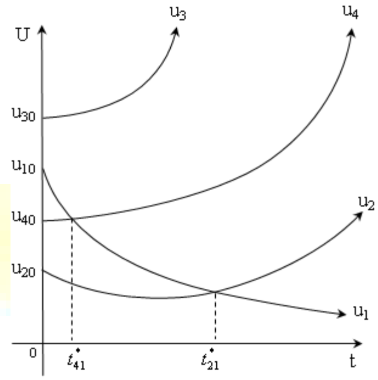



Fig. 5

**Case (ii):** If $u_{30} < u_{20} < u_{40} < u_{10}$ and $a_3 < r_1 < a_4 < a_2$

In this case initially the Prey ($S_1$) dominates $S_4$, the Predator ($S_2$) and the host ($S_3$) of $S_2$ till the time instant $t^*_{41}$, $t^*_{21}$, $t^*_{31}$ respectively and thereafter the dominance is reversed. Also the Predator ($S_2$) dominates the host ($S_3$) of $S_2$ till the time instant $t^*_{32}$ and thereafter the dominance is reversed.

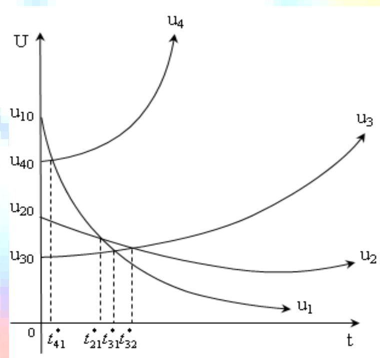



Fig.6

**Case (B):** When $r_1 > 0$ (i.e., when $\frac{a_1}{a_2} > \frac{a_{12}}{a_{22}}$)

The roots $r_1$, $a_3$, $a_4$ are positive and $-a_2$ is negative.

Hence the equilibrium state in **unstable**.

In this case the solutions are same as in case (A) and the solutions are illustrated in figures.

**Case (i):** If $u_{20} < u_{30} < u_{40} < u_{10}$ and $a_3 < a_4 < a_2 < r_1$

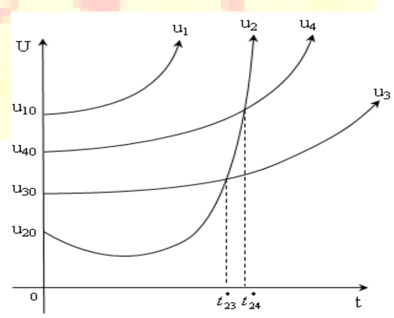
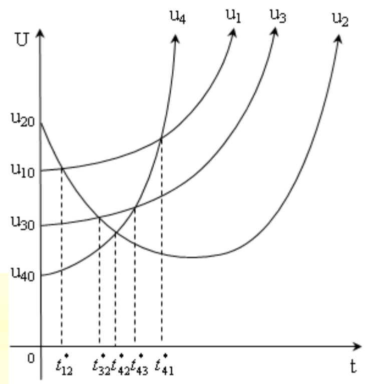



Fig. 7

In this case initially $S_4$ dominates the Predator ($S_2$) till the time instant $t^*_{24}$ and thereafter the dominance is reversed. Also the host ($S_3$) of $S_2$ dominates the Predator ($S_2$) till the time instant $t^*_{23}$ and thereafter the dominance is reversed.




**Case (ii):** If $u_{40} < u_{30} < u_{10} < u_{20}$ and $a_2 < a_3 < r_1 < a_4$

In this case initially the Predator ($S_2$) dominates the Prey ($S_1$), the host ($S_3$) of $S_2$ and $S_4$ till the time instant $t^*_{12}$, $t^*_{32}$, $t^*_{42}$ respectively and thereafter the dominance is reversed. Also the Prey ($S_1$) dominates $S_4$ till the time instant $t^*_{41}$ and the dominance gets reversed thereafter. Similarly the host ($S_3$) of $S_2$ dominates $S_4$ till the time instant $t^*_{43}$ and thereafter the dominance is reversed.

Fig. 8

### 4.4 Stability of the Equilibrium State $E_5$:

Substituting (4.1.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of $u_1$, $u_2$, $u_3$, $u_4$, we get

$$\frac{du_1}{dt} = -a_1 u_1 - \frac{a_{12} a_1}{a_{11}} u_2 \quad (4.4.1) \qquad \frac{du_2}{dt} = q_2 u_2 \quad (4.4.2)$$

$$\frac{du_3}{dt} = a_3 u_3 \quad (4.4.3) \qquad \frac{du_4}{dt} = a_4 u_4 \quad (4.4.4)$$

Here $q_2 = a_2 + \dfrac{a_{21} a_1}{a_{11}}$ (4.4.5)

The characteristic equation of which is

$$(\lambda + a_1)(\lambda - q_2)(\lambda - a_3)(\lambda - a_4) = 0 \quad (4.4.6)$$

The roots $q_2$, $a_3$, $a_4$ are positive and $-a_1$ is negative.

Hence the equilibrium state in **unstable**.

The solutions of the equations (4.4.1) (4.4.2), (4.4.3), (4.4.4) are

$$u_1 = [u_{10} + \frac{a_1 a_{12} u_{20}}{a_{11}(q_2 + a_1)}] e^{-a_1 t} - \frac{a_1 a_{12} u_{20}}{a_{11}(q_2 + a_1)} e^{q_2 t} \quad (4.4.7)$$

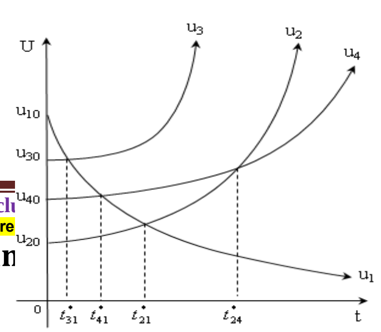
$$u_2 = u_{20} e^{q_2 t} \quad (4.4.8) \qquad u_3 = u_{30} e^{a_3 t} \quad (4.4.9)$$

$$u_4 = u_{40} e^{a_4 t} \quad (4.4.10)$$

The solutions are illustrated in figures.

**Case (i):** If $u_{20} < u_{40} < u_{30} < u_{10}$ and $a_4 < a_1 < q_2 < a_3$

In this case initially the Prey ($S_1$) dominates the host ($S_3$) of $S_2$, $S_4$ and the Predator ($S_2$) till the time instant $t^*_{31}$, $t^*_{41}$, $t^*_{21}$ respectively and thereafter the dominance is reversed. Also $S_4$



dominates the Predator ($S_2$) till the time instant $t^*_{24}$ and thereafter the dominance is reversed.

Fig. 9

**Case (ii):** If $u_{30} < u_{10} < u_{40} < u_{20}$ and $a_4 < a_3 < a_1 < q_2$

In this case initially $S_4$ dominates the host ($S_3$) of $S_2$ till the time instant $t^*_{34}$ and the dominance gets reversed thereafter. Also the Prey ($S_1$) dominates the host ($S_3$) of $S_2$ till the time instant $t^*_{31}$ and thereafter the dominance is reversed.

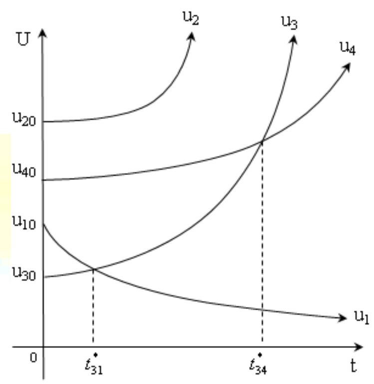



Fig. 10

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